JEE-Main-26-02-2021-Shift-1 (Memory Based) PHYSICS

Question: Find the significant figure in 50000.020×10^{-3} **Options:**

(a) 5
(b) 6
(c) 7
(d) 8
Answer: (d)
Solution:
All the digits in the number part are significant. Therefore, number of significant figures is 8.

Question: A block of mass, 2 kg is kept on a smooth surface as shown. It is being applied by force $\overline{F} = 20\hat{i} + 10\hat{j}$. Find the displacement of the block in 10 seconds.



$$\frac{1}{2} \times a_x \times t^2$$
$$= \frac{1}{2} \times 10 \times 10^2 = 500 \, m$$

Question: A tunnel is dug is the earth as shown. A particle of mass 'm' is released in the tunnel, find the time period of SHM of the particle: - $(\rho: \text{density of earth})$



Options:

(a)
$$T = 2\pi \sqrt{\frac{3}{4\pi\rho G}}$$

(b) $T = 2\pi \sqrt{\frac{3}{2\pi\rho G}}$
(c) $T = 2\pi \sqrt{\frac{3}{\pi\rho G}}$
(d) $T = 2\pi \sqrt{\frac{3}{4\rho G}}$

Answer: (a) Solution:

Time period of a particle along a tunnel dug in earth would be $T = 2\pi \sqrt{\frac{R}{g}}$

$$\Rightarrow T = 2\pi \sqrt{\frac{R}{\left(\frac{4}{3}\pi R\rho\right)}} \quad \left[\because g = \frac{4}{3}\pi R\rho\right]$$
$$\Rightarrow T = 2\pi \sqrt{\frac{3}{4\pi\rho G}}$$

Question: Four solid spheres each of mass m and radius a, have their centers on the vertices of a square whose side is b. Find moment of inertia about the given axis? (Assume spheres are connected using a light frame)

Options:



Answer: (a) Solution:



 $\Rightarrow \text{ M.O.I of sphere (1) and (2) around axis } AB = I_1 = I_2 = \frac{2}{5}ma^2$ $\Rightarrow \text{ M.O.I of sphere (3) and (4) around axis } AB = I_3 = I_4 = \left(mb^2 + \frac{2}{5}ma^2\right)$ (By using Parallel axis theorem)

Total M.O.I =
$$I_1 + I_2 + I_3 + I_4$$

= $m \left(\frac{8}{5} a^2 + 2b^2 \right)$

Question: For an amplitude modulated wave the max voltage is 16 voltage and minimum voltage is 8 volts then if the modulation factor is $x = 10^{-2}$, then x is? **Answer:** 33.33 **Solution:**

$$V_{\text{max}} - V_{\text{min}}$$

$$m = \frac{16}{V_{\text{max}} - V_{\text{min}}}$$
$$= \frac{(16 - 8)}{(16 + 8)} = \frac{8}{24} = \frac{1}{3}$$
$$= 0.3333$$

 $m = 0.33 = 33.33 \times 10^{-2}$ So, x = 33.33

Question: An AC current is given by $i(t) = I_1 \sin wt + I_2 \cos wt$. Find the rms value of current. **Ontions:**

(a)
$$i_{rms} = \frac{\sqrt{I_1^2 + I_2^2}}{2}$$

(b) $i_{rms} = \frac{\sqrt{I_1^2 + I_2^2}}{\sqrt{2}}$
(c) $i_{rms} = \frac{I_1 + I_2}{2}$
(d) $i_{rms} = 2(\sqrt{I_1^2 + I_2^2})$
Answer: (b)
Solution:
 $i(t) = I_1 \sin \omega t + I_2 \cos \omega t$
So, we could write:
 $\Rightarrow i(t) = \sqrt{I_1^2 + I_2^2} \sin(\omega t + \phi)$
[Where $\phi = \tan^{-1}(\frac{I_2}{I_1})$]
 $\Rightarrow i(t) = \sqrt{I_1^2 + I_2^2} \sin(\omega t + \phi)$
So rms value would be
 $i_{rms} = \frac{\sqrt{I_1^2 + I_2^2}}{\sqrt{2}}$

Question: Initially two monoatomic gases are placed in an adiabatic container with insulated fixed partition. The initial pressure, volume and numbers of moles of the two gases are given. Now if the partition is removed. Find the final pressure of mixture.

4.5 lt	5.5 lt
2 atm	3 atm
3 moles	4 moles

Options:

(a) 1 atm

- (b) 2.55 atm
- (c) 5 atm (1) 7
- (d) 7 atm

Answer: (b) Solution: By conservation of Internal energy $n_1\left(\frac{3}{2}RT_1\right) + n_2\left(\frac{3}{2}RT_2\right) = (n_1 + n_2)\frac{3}{2}RT$ $\Rightarrow n_1T_1 + n_2T_2 = (n_1 + n_2)T$ Using (PV = nRT) $\frac{P_1V_1}{R} + \frac{P_2V_2}{R} = \frac{P_fV_f}{R}$ $P_f = \frac{P_1V_1 + P_2V_2}{V_f}$ $= \frac{P_1V_1 + P_2V_2}{(V_1 + V_2)}$

$$=\frac{(4.5)(2)+(5.5)(3)}{10}$$

= 2.55 atm

Question: Two slabs are placed adjacently having thermal resistance $R_1 \& R_2$ their free ends maintained at temperature of $\theta_1 \& \theta_2$ respectively. Find the temperature of junction.



Now, it is given that both the slabs are in series

So, i_H will same for both the slabs. Let the temperature of junction be θ Then,

$$\begin{split} i_{H_1} &= i_{H_2} \\ \frac{\theta_1 - \theta}{R_1} &= \frac{\theta - \theta_2}{R_2} \\ \theta_1 R_2 - \theta R_2 &= \theta R_1 - \theta_2 R_1 \\ \theta R_1 + \theta R_2 &= \theta_1 R_2 + \theta_2 R_1 \\ \theta &= \frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2} \end{split}$$

Question: Find the equivalent resistance between A & B if all resistance are 'R'?



All resistance are R

This circuit satisfies wheats stone bridge condition.



Question: Some drops, each of radius 'r' coalesce to form a large drop of radius 'R'. The surface tension is T. Find the change in surface energy per unit volume. Options:

(a)
$$T\left(\frac{1}{r} - \frac{1}{R}\right)$$

(b) $3T\left(\frac{1}{r} - \frac{1}{R}\right)$
(c) $3T\left(\frac{1}{R} - \frac{1}{r}\right)$
(d) $T\left(\frac{1}{R} - \frac{1}{r}\right)$

Answer: (b)

Solution:

Radius of small drop is r and radius of big drop is R.

When they coalesce, the volume will the same before and after coalesce.

$$\frac{4}{3}\pi R^{3} = n \cdot \frac{4}{3}\pi r^{3} = \text{volume (V)}$$
$$R = n^{\frac{1}{3}}r \quad \dots (1)$$

The surface area of large drop is $4\pi R^2$ and surface area of small drop is $4\pi r^2$ Let

 ΔU – change in surface energy

 ΔA – Change in surface area

T-surface tension Then, $\Delta U = T.\Delta A$ $\Delta U = T.(4\pi R^2 - 4\pi r^2)$

$$\Delta U = \frac{3T}{3} \left(\frac{4\pi R^3}{R} - \frac{4\pi r^3}{r} \right)$$
$$\Delta U = 3T \left(\frac{4\pi R^3}{3} - \frac{4\pi r^3}{3} \right)$$
$$\Delta U = 3T \left(\frac{V}{R} - \frac{V}{r} \right)$$
$$\left| \frac{\Delta U}{V} \right| = 3T \left(\frac{1}{r} - \frac{1}{R} \right)$$

Question: If the force acting on the body moving in circular motion is proportional to r^{-3} , then the time period of its revolution is proportional to **Options:**

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$ (c) r^2 (d) rAnswer: (c) Solution: $F = \frac{mv^2}{r}$ According to question, $F \propto r^{-3}$ Or $F = kr^{-3}$ So, $\frac{mv^2}{r} = kr^{-3}$ $v^2 = \frac{kr^{-2}}{r}$ $v^2 = \frac{kr^{-2}}{m}$ Or $v = \sqrt{\frac{k}{m}} \cdot r^{-1}$ Now, $T = \frac{2\pi r}{v}$ $\therefore T = \frac{2\pi r}{\sqrt{\frac{k}{m}} \cdot r^{-1}}$ So $T \propto r^2$

Question: A wire of length 'L' carries charge 'Q' uniformly distributed along its length. Find the electric field at a distance $\frac{\sqrt{3}L}{2}$ from the wire at point 'A' as shown.



Options:

(a)
$$E_A = \frac{Q}{2\sqrt{3}\pi \in_0 L^2}$$

(b) $E_A = \frac{Q}{\sqrt{3}\pi \in_0 L^2}$
(c) $E_A = \frac{Q}{4\sqrt{3}\pi \in_0 L^2}$
 $2Q$

(d)
$$E_A = \frac{2Q}{\sqrt{3}\pi \in_0 L^2}$$

Answer: (a)

Solution:

Electric field at a distance r from a uniformly charged finite length rod is given by -2KO

$$E = \frac{2\pi g}{r\sqrt{L^2 + 4r^2}}$$

Where L is length of the rod and r is perpendicular distance of point from rod.

So,
$$E = \frac{1}{2\pi \epsilon_0} \cdot \frac{Q}{\sqrt{3L}} \cdot \left(\sqrt{L^2 + 3L^2}\right)$$
$$= \frac{1}{2\sqrt{3\pi \epsilon_0}} \cdot \frac{Q}{L^2}$$

Question: Find the force of attraction between a solid sphere of mass M and a ring of mass 'm' as shown





(a)
$$F = \frac{GMm}{Q\sqrt{3}R^2}$$

(b) $F = \frac{\sqrt{3}GMm}{8R^2}$
(c) $F = \frac{GMm}{8R^2}$
(d) $F = \frac{GMm}{3R^2}$

Answer: (b) Solution:

Electric field intensity on the axis of ring is given by

$$\vec{E}_g = \frac{Gmx}{\left(r^2 + x^2\right)^{\frac{3}{2}}}$$

Here, r = R and $x = \sqrt{3}R$

$$\therefore \vec{E}_g = \frac{Gm\sqrt{3}R}{\left(R^2 + 3R^2\right)^{\frac{3}{2}}} = \frac{\sqrt{3}Gm}{8R^2}$$
$$\vec{F} = M.\vec{E}_g = \frac{\sqrt{3}GMm}{8R^2}$$

Question: A travelling wave is given by $y = -0.21\sin(x+3t)$ where x is in m, t is in seconds, y is in mm. (mass per unit length = 0.135 g/cm). Find the tension in the wire. Options:

(a) T = 0.1215 N(b) T = 20 N(c) T = 15.35 N(d) T = 5 NAnswer: (a) Solution: $y = -0.21 \sin(x + 3t)$

Comparing this equation with $y = A\sin(kx + \omega t)$

 $A = -0.21, k = 1, \omega = 3$ Velocity of wave,

$$v = \frac{\omega}{k} = 3 m / s$$

Now, $v = \sqrt{\frac{T}{\mu}}$

Where μ is mass per unit length of wire.

So,
$$T = \mu v^2$$

= $\left[\frac{0.135 \times 10^{-3}}{10^{-2}}\right] \times (3)^2$
= 0.1215 N

Question: If W= work done, T = Temperature, K_B = Boltzmann constant, x = Displacement

& $W = \alpha \beta^2 e^{-\frac{x^2}{\alpha K_B T}}$, then find dimensions of β ? **Options:** (a) $\left[M^1 L^2 T^{-2} \right]$ (b) $\left[M^{-1} L^2 T^{-2} \right]$ (c) $\left[M^1 L^1 T^{-2} \right]$ (d) None of these **Answer:** (c) **Solution:** $W = \alpha \beta^2 e^{-\frac{x^2}{\alpha K_B T}}$ So, dimensions of $\left[W \right] = \left[M L^2 T^{-2} \right]$ $\left[x \right] = \left[L \right]$ $\left[K_B \right] = \left[M L^2 T^{-2} K^{-1} \right]$

[T] = [K]

Now we know that power of e is constant i.e. dimensionless

So,
$$\left[\frac{x^2}{\alpha K_B T}\right] = \left[M^0 L^0 T^0 K^0\right]$$

 $\therefore [\alpha] = \left[M^{-1} T^2\right]$
To find dimensions of β
 $[W] = [\alpha] \left[\beta^2\right]$
 $\left[\beta^2\right] = \frac{\left[ML^2 T^{-2}\right]}{\left[M^{-1} T^2\right]} = \left[M^2 L^2 T^{-4}\right]$
 $\Rightarrow [\beta] = \left[ML T^{-2}\right]$

Question: When two capacitors C_1 and C_2 are in parallel, the equivalent capacitance is C_{11} . When the same two are in series the equivalent is C_s . The ratio of $C_{11}: C_s$ is 15:4. Find

$$\frac{C_2}{C_1}?$$

Answer: (Bonus) Solution:

$$\frac{C_{II}}{C_{s}} = \frac{15}{4}$$
$$\frac{C_{1} + C_{2}}{\left(\frac{C_{1}C_{2}}{C_{1} + C_{2}}\right)} = \frac{15}{4}$$
$$4(C_{1} + C_{2})^{2} = 15C_{1}C_{2}$$

This question will not lead to a real solution.

JEE-Main-26-02-2021-Shift-1 (Memory Based) CHEMISTRY

Question: Ozone in troposphere prevents us from? **Options:**

- (a) Cause photochemical smog
- (b) Protects from UV radiation
- (c) Protects from X-ray
- (d) Protects from greenhouse effect

Answer: (b)

Solution: Ozone in troposphere, absorbs radiation especially UV radiations and protects humans and other living species from harmful effects of UV rays emitting from sun.

Question: Statement 1: O-nitrophenol is steam volatile due to H- bonding Statement 2: O-nitrophenol has higher melting point due to H-Bonding

Options:

- (a) Both statements are true
- (b) S₁ is is true, S₂ is false
- (c) S₁ is false, S₂ is true
- (d) Both are false

Answer: (b)

Solution: O-nitrophenol has intramolecular H-bonding while p-nitrophenol has intermolecular H-bonding. Due to this, molecules of o-nitrophenol are weakly held together and therefore steam volatile.

- O nitrophenol's melting point = 216° C
- P nitrophenol's melting point = 279° C

Question: Deficiency of which vitamin helps in delaying blood clotting?

Options:

- (a) Vitamin B
- (b) Vitamin K
- (c) Vitamin C
- (d) Vitamin E

Answer: (b)

Solution: Blood clotting is delayed or prevented because vitamin K is unavailable to act as an essential element (a cofactor) for the coagulation (clotting).

Question:



A and B are? **Options:**

(a)





(d)



Answer: (a) Solution:



Question: What is the structure of neoprene?

Options: (a) $\begin{pmatrix} CH_2 - C = CH - CH_2 \\ CI \end{pmatrix}_n^{(b)} \\ \begin{pmatrix} CH_2 - C = C < CH_2 \\ H \end{pmatrix}_n^{(c)} \\ \begin{pmatrix} O & H \\ C - (CH_2)_5 - N \end{pmatrix}_n^{(c)} \\ \begin{pmatrix} d \\ CF_2 - CF_2 \end{pmatrix}_n^{(c)} \\ \end{pmatrix}$

Answer: (a) Solution:

$$\begin{pmatrix} CH_2 - C = CH - CH_2 \\ Cl \end{pmatrix}_n$$

Question: S₁: Dipole-dipole interactions are the only non-covalent interaction that lead to hydrogen bonding.

S₂: Fluorine is the most electronegative element and hydrogen bonding in HF is symmetrical. **Options:**

(a) Both statements are true

(b) S₁ is true, S₂ is false

(c) S_1 is false, S_2 is true

(d) Both are false

Answer: (c)

Solution: ion-dipole interaction can also lead to H-bonding For example:

$$\begin{array}{c} KF + HF \rightarrow KHF_{2} \\ \downarrow \\ HF_{2}^{\odot} \end{array}$$

Question: Number of Bridging CO in Mn2(CO)10



Question: How many faraday of electricity required to reduce 5 mole of $MnO4^{-}$ as per the given reaction :

 $MnO4^{-} + 16H^{+} + 5e^{-} \longrightarrow Mn^{2+} + 8H_2O$

Options:

(a) 5 F

- (b) 20 F
- (c) 25 F
- (d) 10 F

Answer: (c)

Solution:

 $MnO_4^- + 5e^- \rightarrow Mn^{2+}$

For, 1 mole reduction $\Rightarrow 5F$ electricity required For, 5 mole reduction $\Rightarrow 5 \times 5 = 25F$ electricity required

Question:

The product is: $Amine \xrightarrow{Hinsberg's \ reagent} \xrightarrow{Product}_{(insoluble \ in \ NaOH)}$

That amine can also be prepared by ammonolysis of ethyl chloride

Options:

(a)

$$\overset{H}{\searrow^{N}}$$

(b)









Question:

$$C_4H_8Cl_2(A) \xrightarrow{Hydrolysis} C_4H_8O(B)$$

B forms oxime with NH_2OH but does not give Tollen's test.

Compound (A) and (B) are respectively:

Options:

(a) 2,2 – Dichlorobutane and 2 - Butanone

(b) 2,2 – Dichlorobutane and 2 – Butanal

(c) 1,1 – Dichlorobutane and 2 – Butanal

(d) 1,2 – Dichlorobutane and 2 – Butanone

Answer: (a)

Solution:



Question: Match I.E with the elements of configuration given below:

(a) $1s^2 2s^2$	(i) 810
(b) $1s^2 2s^2 2p^4$	(ii) 899
(c) $1s^2 2s^2 2p^3$	(iii) 1300
(d) $1s^2 2s^2 2p^1$	(iv) 1490
Options:	
$(a) (a) \rightarrow (i) ; (b) - (iii)$	
$(b) (a) \rightarrow (ii); (b) - (iii)$	
$(c) (a) \rightarrow (i); (b) - (iv)$	
$(d) (a) \rightarrow (iv); (b) - (iii)$	
Answer: (b)	
Solution, Dubog highou I D	then D. It a

Solution: Br has higher I.E than B. It can Bl, the last electron is in s-orbital while in case of B it is in p-orbital removal of electron. From s-orbital requires more energy than the electron from p-orbital

 $2s^22p^3$ configuration is more stable than $2s^22p^4$ due to half-filled p-subshell. So Cl I.E is more as it is filled to remove electron from stable half-filled configuration.

Question:

Compound + dil H₂SO₄ \rightarrow X H₂SO₄ \downarrow K₂Cr₂O₇ Y What are X and Y respectively? Options: (a) SO₂ , Cr₂O₃ (b) SO₃ , Cr₂O₃ (c) SO₃ , Cr₂(SO₄)₃ (d) SO₂ , Cr₂(SO₄)₃ Answer: (d) Solution:

 $3SO_{2} + K_{2}Cr_{2}O_{7} + H_{2}SO_{4} \rightarrow Cr_{2}(SO_{4})_{3} + K_{2}SO_{4} + H_{2}$

Question: Oxidation number of Cr in product, when $Cr_2O_7^{2-}$ reacted with OH⁻

Options:

(a) +4 (b) +3 (c) +6 (d) 0 **Answer: (c)**

Solution: $Cr_2O_7^{2^-} + 2OH^- \rightarrow 2CrO_4^{2^-} + H_2O$ $Cr_2O_7^{2^-} \Rightarrow \text{Oxidation state} = +6$ $CrO_4^{2^-} \Rightarrow \text{Oxidation state} = +6$

Question: A gas law obeys $P(V_m - b) = RT$, and dz/dp = x b / (RT), what is the value of X **Options:**

(a) 2 (b) 0.5 (c) 2.5 (d) 1 Answer: (d) Solution: $Z = \frac{V_R}{V_i} \rightarrow \text{real gas } V_m$ $PV_{m^{-}} pb = RT$ $V_m = \frac{RT + pb}{P}$ $Z = \frac{RT + pb}{pV_i}$ $=\frac{RT+pb}{RT}\{\because PV_m=RT\}$ $Z = 1 + \frac{pb}{RT}$ $\frac{dz}{dp} = \frac{b}{RT} \therefore x = 1$ **Question:** Match the following: (a) Cryolite (i) Sn (b) Cassiterite (ii) Chloride (iii) Zn (c) Calamine (d) Carnallite (iv) Fluoride **Options:** (a) (a) \rightarrow i; (b) \rightarrow ii; (c) \rightarrow iii; (d) \rightarrow iv (b) (a) \rightarrow iv; (b) \rightarrow i; (c) \rightarrow iii; (d) \rightarrow ii (c) (a) \rightarrow ii; (b) \rightarrow i; (c) \rightarrow iii; (d) \rightarrow iv (d) (a) \rightarrow iv; (b) \rightarrow iii; (c) \rightarrow ii; (d) \rightarrow i Answer: (b) Solution: Cryolite Na_3AlF_6 Cassiterite SnO₂ $ZnCO_3$ Calamine

Carnallite

 $KCl.MgCl_2.6H_2O$

Question:



Options:







Question: Which of the following doesn't form MO₂ type oxide? **Options:**

(a) Pr

(b) Dy

(c) Nd

(d) Yb

Answer: (d)

Solution: oxidation states of +2 & +3 and is not stable in +4 oxidation state so it does not form MO₂ type oxides

Question: $\Delta S = 27 \text{ J/K-mol}$ (values not confirmed)

 $\Delta H = 80 \text{ KJ/mol}$

What is the value of temperature at which reaction becomes spontaneous?

Options:

(a) 2960 K (b) 2964 K (c) 2952 K (d) 2949 K Answer: (b) Solution: $\Delta G = \Delta H - T \cdot \Delta S$ For spontaneity, $\Delta G < 0$ $\therefore \Delta G = \Delta H - T \cdot \Delta S$ $\Delta H - T \cdot \Delta S < 0$ $\Delta H < T \Delta S$ $T > \frac{\Delta H}{\Delta S}$ $T > \frac{\Delta H}{\Delta S}$ $T > \frac{80,000}{27}$ T > 2963K

Question: Half heroult's process is used for

Options:

- (a) extraction of Au
- (b) extraction of Zn(c) extraction of Ae
- (d) extraction of Ag

Answer: (c)

Solution:

Half heroult's process is used for extraction of Ae where carbon anode is oxidised to Co and CO_2 . Al⁺³ is reduced at cathode to form Al.

 $2Al_2O_3 + 3C \rightarrow 4Ae + 3CO_2$ At Cathode : Al⁺³ + 3e⁻ --- \rightarrow Al At Anode $\rightarrow C + O^{2-} \rightarrow CO + 2e^ C + 2O^{2-} \rightarrow CO_2 + 4e^-$

Question: Orbital having 2 radial and 2 angular nodes is

Options:

(a) 5d (b) 3d (c) 4f (d) 5f Answer: (a) Solution: No. of Radial nodes : n - 1 - 1No. of Angular nodes : 2 $\therefore 1 = 2; \rightarrow d$ -orbital $n - \ell - 1 = 2$ n - 2 - 1 = 2 n = 5 $\therefore 5d$

Question: Compound A is used as an oxidising agent to amphoteric in nature. It is part of the lead storage battery compound A is

Options:

- (a) *PbO*
- (b) $Pb_{3}O_{4}$
- (c) $2PbO, PbO_2$
- (d) PbO_2

Answer: (d)

Solution: In lead storage battery, PbO_2 is the oxidising agent that is amphoteric in nature as it reacts with both acids and bases So A is PbO_2

Question: What is the major product of the following reaction?



It is free-radical substitution reaction of alkanes, so bromination takes place of benzylic carbon.

Question: Statement – I: Chloroform and aniline is separated by simple distillation. Statement – II : When we separate water and aniline by steam distillation aniline boils below its boiling point.

Options:

- (a) Statement I is true, Statement II is false
- (b) Statement I is false, Statement II is true
- (c) Statement I, II both are true
- (d) Statement I, II both are false

Answer: (c)

Solution: Simple distillation may be used when the boiling points of two liquids are significantly different from each other or to separate liquids from solids or nonvolatile components. In simple distillation, a mixture is heated to change the most volatile component from a liquid into vapor.

Question: Which statement is false? **Options:**

(a) Kjeldal method is used for estimation of nitrogen.

(b) Carius tube is used for estimation of sulphur

(c) Carius tube is used for estimation of nitrogen

(d) Phosphoric acid is precipitated by adding magnesia mixture on yields $Mg_2P_2O_7$

Answer: (c)

Solution: Carius method is used for quantitative estimation of halogen, sulphur and phosphorous.

JEE-Main-26-02-2021-Shift-1 (Memory Based) MATHEMATICS

Question: x - y = 0, 2x + y = 6, x + 2y = 3, triangle formed by these lines is

Options:

- (a) right angled(b) equilateral(c) isosceles
- (d) none

Answer: (c) Solution:



 $BC = \sqrt{2}$ $AC = \sqrt{5}$

Hence it is isosceles

Question:
$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+3^x} dx = ?$$
Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^{2} x}{1 + 3^{x}} dx \qquad \dots(i)$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^{2} x}{1 + 3^{-x}} dx$$

$$\left[using \int_{a}^{b} f(x) dx = \int_{a}^{b} f(a + b - x) dx \right]$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{3^{x}}{3^{x} + 1} \cos^{2} x dx \qquad \dots(ii)$$

$$(i) + (ii)$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^{2} x dx = 2\int_{0}^{\frac{\pi}{2}} -\cos^{2} x dx$$

$$I = \int_{0}^{\frac{\pi}{2}} \cos^{2} x dx \qquad \dots(iii)$$

$$I = \int_{0}^{\frac{\pi}{2}} \sin^{2} x dx \qquad \dots(iii)$$

$$(iii) + (iv)$$

$$2I = \int_{0}^{\frac{\pi}{2}} 1 dx = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Question: The sum of the infinite values of $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} \dots \infty$

Options:

(a) (b) (c) (d) Answer: () Solution: $S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \dots \infty \qquad \dots (i)$ $\frac{1}{3}S = \frac{1}{3} + \frac{2}{2^2} + \frac{7}{3^3} + \dots \infty \qquad \dots (ii)$ (i) - (ii) $\frac{2}{3}S = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \dots$ $= \frac{4}{3} + \frac{\frac{5}{3^2}}{1 - \frac{1}{3}}$ $\frac{2}{3}S = \frac{4}{3} + \frac{5}{6}$ $\implies S = 2 + \frac{5}{4} = \frac{13}{4}$

Question: Growth of bacteria is directly proportional to number of bacteria. At t = 0, number of bacteria = 1000 and after 2 hours population is increased by 20%. After this population

becomes 2000 where $t = \frac{k}{\ln\left(\frac{6}{5}\right)}$. Find value of $\left(\frac{k}{\ln 2}\right)^2$. **Options:** (a) 4 (b) 8 (c) 16 (d) 32 Answer: (a) Solution: Let, number of bacteria = N $\frac{dN}{dt} \propto N$ $\frac{dN}{dt} = aN$ $\int \frac{dN}{N} = a \int dt$ $\log N = at + C$ At t = 0, $N = 1000 \Rightarrow C = \log 1000$ At t = 2, $N = 1 \cdot 2(1000) = 1200$ $\Rightarrow \log 1200 = 2a + \log 1000$

$$\Rightarrow 2a = \log\left(\frac{1200}{1000}\right)$$
$$\Rightarrow a = \frac{1}{2}\log\left(\frac{6}{5}\right)$$
At $t = \frac{k}{\ln\left(\frac{6}{5}\right)}$, $N = 2000$
$$\Rightarrow \log 2000 = \frac{1}{2}\log\left(\frac{6}{5}\right) \cdot \frac{k}{\log\left(\frac{6}{5}\right)} + 1000$$
$$\Rightarrow \log\left(\frac{2000}{1000}\right) = \frac{k}{2}$$
$$\Rightarrow \frac{k}{\log 2} = 2$$
$$\Rightarrow \left(\frac{k}{\log 2}\right)^2 = 4$$

Question: If $|f(x) - f(y)| \le (x - y)^2 \forall x, y \in R$ and f(0) = 1, then Options: (a) $f(x) < 0 \forall x \in R$ (b) $f(x) > 0 \forall x \in R$ (c) $f(x) = 0 \forall x \in R$ (d) Answer: (c) Solution: $|f(x) - f(y)| \le (x - y)^2$ $\Rightarrow |f(x+h) - f(x)| \le (x+h-x)^2$ $|f(x+h) - f(x)| \le h^2$ $-h^2 \le f(x+h) - f(x) \le h^2$ $-h \le \frac{f(x+h) - f(x)}{h} \le h$ $\Rightarrow \lim_{h \to 0} (-h) \le \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \le \lim_{h \to 0} h$

$$\Rightarrow 0 \le f'(x) \le 0$$

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow f(x) = \text{constant}$$

$$\because f(0) = 1$$

$$\therefore f(x) = 1 \quad \forall \ x \in R$$

Question: A coin is flipped 'n' times. Probability of seven heads is equal to probability of nine heads. What is probability of getting exactly 2 heads **Options:**

(a) $\frac{15}{2^{13}}$ (b) $\frac{15}{2^{16}}$ (c) $\frac{15}{2^{12}}$

(d) none Answer: (a) Solution: Let n be total no. of tosses

x-number of times head occurs

$$\therefore p = \frac{1}{2} = q$$

$$P(7) = P(9)$$

$${}^{n}C_{7}\left(\frac{1}{2}\right)^{7}\left(\frac{1}{2}\right)^{n-7} = {}^{n}C_{9}\left(\frac{1}{2}\right)^{9}\left(\frac{1}{2}\right)^{n-9}$$

$${}^{n}C_{7} = {}^{n}C_{9}$$

$$\implies n = 7 + 9 = 16$$

$$P(2) = {}^{16}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{14} = \frac{16 \times 15}{2 \cdot 2^{16}} = \frac{15}{2^{13}}$$

Question: How many 7 digit numbers can be made using 1, 2, 3 such that its sum of digits is 10

Options:

(a) 77

(b) 76

- (c) 78 (d) 80
- (d) 80

Answer: (a) Solution:

2 ways of having sum 10 using these digits

{3, 2, 1, 1, 1, 1, 1} or {2, 2, 2, 1, 1, 1, 1}

$$\frac{7!}{5!} + \frac{7!}{3!4!}$$

= $42 + \frac{7 \times 6 \times 5}{6}$
= 42 + 35
= 77

Question: The value of $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]} dx =$ **Options:** (a) 100(e-1)(b) 100e

- (c) 100(1+e)
- (d) 100(1-e)

Answer: (a)

Solution:

$$\int_{0}^{n_{1}} f(x) dx = n \int_{0}^{1} f(x) dx$$

Here
$$x - [x] = \{x\}$$

 $\{x\}$ has period 1

$$I = \int_{0}^{100} e^{\{x\}} dx$$

= $\int_{0}^{1} e^{x} + \int_{1}^{2} e^{x-1}$
= $100 \int_{0}^{1} e^{x} dx = 100 [e^{x}]_{0}^{1}$
= $100 (e^{1} - 1)$
= $100 (e - 1)$



Options:

(a) $24\sqrt{3}$ (b) $24\sqrt{2}$ (c) $26\sqrt{3}$

(d) $26\sqrt{2}$ Answer: (a) Solution:



$$\Rightarrow x^{2} + 5.5^{2} = 6.5^{2}$$

$$x^{2} = 12$$

$$x = 2\sqrt{3}$$
Area $\Delta PQB = \frac{1}{2} \times 12 \times 2 \times 2\sqrt{3}$

$$= 24\sqrt{3}$$

Question: $3\cos x + 2\sin x = k + 1$ then set integral values of *k*. **Options:** (a) (b) (c) (d) Answer: () Solution: $-\sqrt{3^2 + 2^2} \le k + 1 \le \sqrt{3^2 + 2^2}$ $\sqrt{13} - 1 \le k \le \sqrt{13} - 1$ So, integral values of k are

-4, -3, -2, -1, 0, 1, 2

Question: $\frac{\sin^{-1}(x)}{a} = \frac{\cos^{-1}(x)}{b} = \frac{\tan^{-1}(y)}{c}$ value of $\left(\frac{\pi c}{a+b}\right)$ **Options:** (a) $\frac{1-y}{1+y}$ (b) $\frac{1-y^2}{1+y^2}$ (c) $\frac{2y}{1+y^2}$ (d) Answer: (b) Solution: $\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\tan^{-1} y}{c}$ $\Rightarrow \frac{\sin^{-1} x + \cos^{-1} x}{a+b} = \frac{\tan^{-1} y}{c}$ $\Rightarrow \frac{\tan^{-1} y}{x} = \frac{\pi}{2(a+b)}$ $\Rightarrow 2 \tan^{-1} y = \frac{\pi c}{(a+b)}$ $\Rightarrow \cos\left(\frac{\pi c}{a+b}\right) = \cos\left(2\tan^{-1}y\right)$ $=\frac{1-\tan^{2}(\tan^{-1}y)}{1+\tan^{2}(\tan^{-1}y)}$ $=\frac{1-y^2}{1+y^2}$

Question: $e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x$ Options: (a) (b) (c) (d) Answer: () Solution:

 $e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x$ Put $e^{\sin y} = t \implies e^{\sin y} \cos \frac{dy}{dx} = \frac{dt}{dx}$ So, $\frac{dt}{dx} + t \cos x = \cos x$ $I.F = e^{\int \cos x \, dx} = e^{\sin x}$ Hence, solution is $t(e^{\sin x}) = \int \cos x \cdot e^{\sin x} \, dx + c$ $e^{\sin y} e^{\sin x} = e^{\sin x} + c$ $e^{\sin y} = 1 + ce^{-\sin x}$

Question:
$$\lim_{h \to 0} 2 \left(\frac{\sqrt{3} \sin\left(h + \frac{\pi}{6}\right) - \cos\left(h + \frac{\pi}{6}\right)}{h\left(\sqrt{3} \cosh - \sinh\right)} \right)$$

Options:

(a) (b) (c) (d) Answer: () Solution: $\left(\frac{\sqrt{3}}{2}\sin\left(h+\frac{\pi}{6}\right)-\frac{1}{2}\cos\left(h+\frac{\pi}{6}\right)\right)$

$$\lim_{h \to 0} 2 \cdot \frac{\left(\frac{-2}{2}\sin\left(h + \frac{\pi}{6}\right) - \frac{1}{2}\cos\left(h + \frac{\pi}{6}\right)\right)}{h\left(\frac{\sqrt{3}}{2}\cosh - \frac{1}{2}\sinh\right)}$$
$$\lim_{h \to 0} 2 \cdot \frac{\left(\cos\frac{\pi}{6}\sin\left(h + \frac{\pi}{6}\right) - \sin\frac{\pi}{6}\cos\left(h + \frac{\pi}{6}\right)\right)}{h\left(\sin\frac{\pi}{3}\cos h - \cos\frac{\pi}{3}\sin h\right)}$$

$$\lim_{h \to 0} 2 \cdot \frac{\sin\left(h + \frac{\pi}{6} - \frac{\pi}{6}\right)}{h \cdot \sin\left(\frac{\pi}{3} - h\right)}$$
$$\lim_{h \to 0} \frac{2 \cdot 1}{\sin\frac{\pi}{3}} = \frac{2}{\sqrt{3}} \cdot 2 = \frac{4}{\sqrt{3}}$$

Question: Find maximum value of term independent of 't' in $\left(tx^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$, $x \in (0, 1)$

(a) (b) (c) (d) Answer: () Solution: ${}^{10}C_{-}\left(tx^{\frac{1}{5}}\right)^{10-r}(1-r)^{\frac{r}{10}}t^{-r}$

For term Independent of t
$$10-2r=0$$

$$T_{6} = {}^{10}C_{5} t^{5} \frac{x^{1} \times (1-x)^{\frac{1}{2}}}{t^{5}}$$

Let $T_{6} = f(x) = {}^{10}C_{5} x \sqrt{1-x}$
 $f'(x) = {}^{10}C_{5} \left(\sqrt{1-x} - \frac{x}{2\sqrt{1-x}}\right)$
 $f'(x) = {}^{10}C_{5} \left(\frac{2-2x-x}{2\sqrt{1-x}}\right)$
 $f'(x) = 0 \Longrightarrow x = \frac{2}{3}$
 $T_{6} = {}^{10}C_{5} \frac{2}{3} \sqrt{\frac{1}{3}}$

Question: $\int_{0}^{\pi} |\sin 2x| dx = ?$ Answer: 2.00 Solution: Let 2x = t



Question: If A is 2 × 2 symmetric matrices with integer elements. $tr(A^2) = 1$. Find the

number of such matrices of A Answer: 4.00 Solution:

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$
$$A^{2} = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} a^{2} + b^{2} & ab + bc \\ ab + bc & b^{2} + c^{2} \end{bmatrix}$$
$$tr(A^{2}) = 1$$
$$\Rightarrow a^{2} + b^{2} + b^{2} + c^{2} = 1$$
$$a^{2} + c^{2} + 2b^{2} = 1$$
$$\therefore a, b, c \in I$$
$$\therefore b = 0$$
$$\Rightarrow a^{2} + c^{2} = 1$$
Now, (a, c) can be $(1, 0), (-1, 0), (0, 1), (0, -1)$

Hence, number of such matrices A = 4

Question: The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ Answer: -2.00 Solution: $R_3 \to R_3 - R_2, R_2 \to R_2 - R_1$ $= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 2(a+3) & 1 & 0 \end{vmatrix}$ = 2a+4-2a-6=-2

Question: Find area bounded by y = ||x-1|-2| with *x*-axis

Answer: 4.00 Solution:







Area
$$=\frac{1}{2} \times 2 \times 4 = 4$$

Question: Find number of solutions of $\sqrt{3}\cos^2 x = (\sqrt{3} - 1)\cos x + 1$; $x \in \left[0, \frac{\pi}{2}\right]$

Answer: 1.00

Solution:

$$\sqrt{3}\cos^{2} x = (\sqrt{3} - 1)\cos x + 1$$

$$\sqrt{3}\cos^{2} x - (\sqrt{3} - 1)\cos x - 1 = 0$$

$$\sqrt{3}\cos^{2} x - \sqrt{3}\cos x + \cos x - 1 = 0$$

$$(\sqrt{3}\cos x + 1)(\cos x - 1) = 0$$

$$\cos x = 1 \text{ or } \cos x = -\frac{1}{\sqrt{3}} \text{ (rejected)}$$

$$\Rightarrow \cos x = 1 \qquad \left(\because x \in \left[0, \frac{\pi}{2}\right]\right)$$

$$x = 0$$

Hence, number of solution is 1

Question: If α , β , γ are roots of $x^3 - 2x^2 + 2x - 1 = 0$. Find $\alpha^{162} + \beta^{162} + \gamma^{162}$ Answer: 3.00 Solution: $(x-1)(x^2 - x + 1) = 0$ $\alpha = 1, \beta = -\omega, \gamma = -\omega^2$ $1^{162} + (-\omega)^{162} + (-\omega^2)^{162} = 1 + 1 + 1 = 3$